The characteristic equation of the reduced order estimator is then given by,

$$\det(s\mathbf{I} - \mathbf{A}_{bb} + \mathbf{L}\mathbf{A}_{ab}) = s^2 - l_1 s - l_2 = 0.$$

The desired characteristic equation for the reduced order estimator poles is

$$\alpha_e(s) = (s + 0.1)^2 = s^2 + 0.2s + 0.01.$$

Thus, $l_1 = -0.2$, and $l_2 = -0.01$. This result can be verified using MATLAB's acker command.

Problems and Solutions for Section 7.8: Compensator Design: Combined Control Law and Estimator

48. A certain process has the transfer function $G(s) = 4/(s^2 - 4)$.
a) Find $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ for this system in observer canonical form.
b) If $u = -\mathbf{K}\mathbf{x}$, compute $\mathbf{K}$ so that the closed-loop control poles are located at $s = -2 \pm 2j$.
c) Compute $\mathbf{L}$ so that the estimator-error poles are located at $s = -10 \pm 10j$.
d) Give the transfer function of the resulting controller (for example, using Eq. (7.177)).
e) What are the gain and phase margins of the controller and the given open-loop system?

Solution:

(a) From the transfer function, we can read off the elements that will give observer canonical form,

$$\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{A}_o\mathbf{x} + \mathbf{B}_o u, \\
y &= \mathbf{C}_o\mathbf{x}, \\
\mathbf{A}_o &= \begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix},\ \mathbf{B}_o = \begin{bmatrix} 0 \\ 4 \end{bmatrix},\ \mathbf{C}_o = \begin{bmatrix} 1 & 0 \end{bmatrix}.
\end{aligned}$$

(b) With $u = -[k_1\ k_2][x_1\ x_2]^T$, we want to achieve the following closed-loop characteristic equation:

$$\alpha_c(s) = (s + 2 + 2j)(s + 2 - 2j) = s^2 + 4s + 8 = 0.$$

From $\det(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K}) = 0$, we obtain,

$$s^2 + 4k_2 s + 4k_1 - 4 = 0.$$

Comparing the coefficients yields $k_1 = 3$, and $k_2 = 1$. This result can be verified using MATLAB's place command.

(c) The estimator roots are determined by the equation $\alpha_e(s) = 0$. We want to find l_1 and l_2 such that,

$$\alpha_e(s) = (s + 10 + 10j)(s + 10 - 10j) = s^2 + 20s + 200.$$

$$\begin{aligned}
\alpha_e(s) &= \det(s\mathbf{I} - \mathbf{A} + \mathbf{L}\mathbf{C}) \\
&= \det\left(\begin{bmatrix} s & -1 \\ -4 & s \end{bmatrix} + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix}\right) \\
&= \det\begin{bmatrix} s + l_1 & -1 \\ -4 + l_2 & s \end{bmatrix} = s^2 + l_1 s + l_2 - 4.
\end{aligned}$$

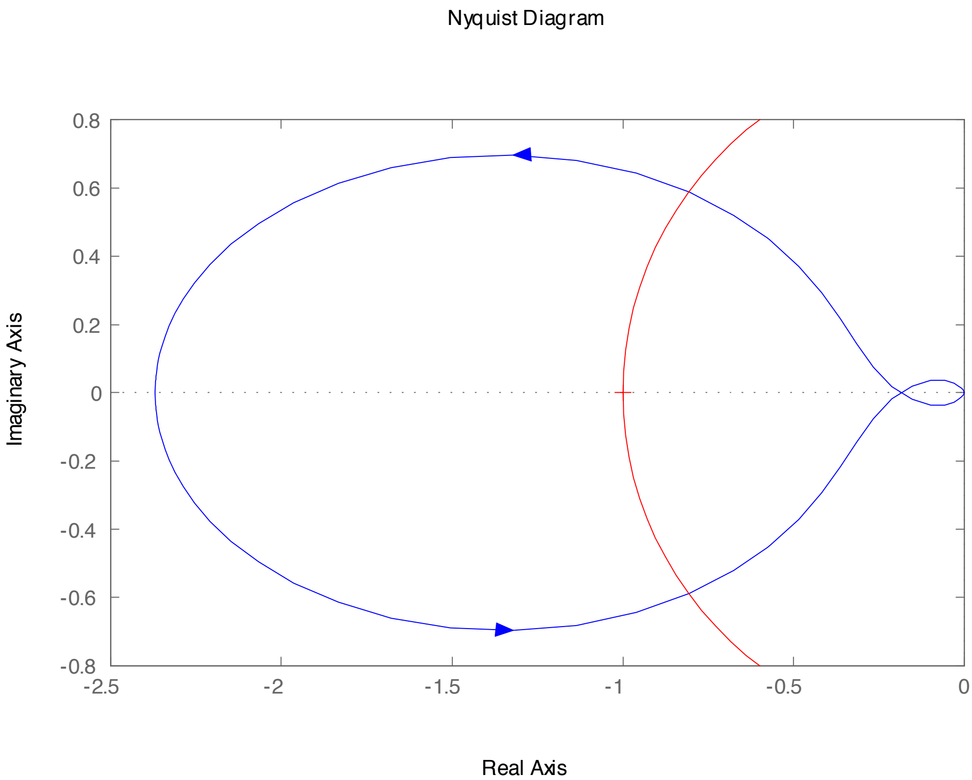
Comparing the coefficients yields $l_1 = 20$, $l_2 = 204$. This result can be verified using MATLAB's place command.

(d) The transfer function of the resulting compensator is,

$$\begin{aligned} D_c(s) &= \frac{U(s)}{Y(s)} = -\mathbf{K}(s\mathbf{I} - \mathbf{A} + \mathbf{BK} + \mathbf{LC})^{-1}\mathbf{L}, \\ &= -\begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} s+20 & -1 \\ 212 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 20 \\ 204 \end{bmatrix} = \frac{-264s - 692}{s^2 + 24s + 292}. \end{aligned}$$

This result can be verified using MATLAB's ss2tf command.

(e) The next figure shows the Nyquist plot generated by MATLAB (using the nyquist command), note that there is both a positive and negative gain margin. The Nyquist plot has a positive gain margin of 0.4220 (i.e., the gain can be increased by $1/0.422 = 2.37$) and a negative margin of 5.46 (i.e., the gain can be decreased by $1/5.46 = 0.183$) before the number of encirclements of the -1 point changes.



Nyquist plot for Problem 7.48.

49. The linearized longitudinal motion of a helicopter near hover (Fig. 7.97) can be modeled by the